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Theory I, Assignment 2

`http://lak.informatik.uni-freiburg.de/lak_teaching/ss_10/theory1.php`

Submission: hand in by 20. May. 2010, before 4:00 p.m.

- The solutions should be submitted in English.
- You are encouraged to work in groups of two, and submit one solution.
- Your solution should be delivered to the lockbox in building 051 floor 0.

Exercise 2.1 - AVL Trees [Points: 2+2+1]

Show that in an AVL tree t with F_k leaves (where F_k is a Fibonacci number) and $k \geq 7$ the *internal path length* $l(t) \leq F_k \cdot (k-4)$. Proceed as follows:

1. Determine the maximum height h of an AVL tree with F_k leaves. Explain informally why in this case an AVL tree with maximum height also has the maxi-mun internal path length.
2. Using induction, show the above inequality.
3. What does this tell us about the average search path length $D(t)$ in such a tree (in terms of its height)?

Exercise 2.2 - Hashing: chaining [Points: 4]

Insert the keys $8, 12, 15, 16, 19, 38, 27, 5, 21, 49, 65, 42$ into a hash table with collisions resolved by chaining. Let the table have 15 slots and let the hash function be $h(k) = k \mod 15$. Show the resulting table.

Exercise 2.3 - Hashing: open addressing [Points: 2+2+2]

Consider an empty hash table of size 15. Insert the following keys

$$8, 12, 15, 16, 19, 38, 27, 5, 21, 49, 65, 42$$

using $h(k) = k \mod 15$ and:

1. Linear probing.
2. Quadratic probing.
3. Double hashing with $h'(k) = 1 + (k \mod 13)$.

Give the resulting tables.

Exercise 2.4 - Universal Hashing [Points: 5]

Let $U = \{0, \ldots, N-1\}$, where N is 49 and m is 35. Let $a_i = 42 \cdot i$ and $b_i = 28 \cdot i$. Now consider the following class of hash functions.

$$\mathcal{H} = \{h_i(k) = ((a_i \cdot k + b_i) \mod N) \mod m\} \text{ for } i \in \{1, \ldots, N(N-1)\}$$

Is $\mathcal{H}$ universal? Prove your answer.