
Theory I, Assignment 4

http://lak.informatik.uni-freiburg.de/lak_teaching/ss_10/theory1.php

Submission: hand in by **24. Jun. 2010, before 4:00 p.m.**

- The solutions should be submitted in English.
- You are encouraged to work in groups of two, and submit one solution.
- Your solution should be delivered to the lockbox in building 051 floor 00.

Exercise 4.1 - Edit distance

[Points: 3+1+1]

Consider two strings $A = ABRACADABRA$ and $B = CHUPACABRAS$.

1. Show the corresponding trace graph for transforming A into B . For each node draw only the feasible edges (i.e. edges that lead to the corresponding minimum value of the node).
2. Mark an optimal trace (i.e. an optimal path in the trace graph).
3. Specify the corresponding sequence of edit operations and $D(A, B)$.

Exercise 4.2 - Term induction

[Points: 3+2]

Let s, t be terms and p, q be strings over the natural numbers. Prove the following propositions:

1. If $pq \in \mathcal{P}os(s)$, then $s|_{pq} = (s|_p)|_q$.
2. If $p \in \mathcal{P}os(s)$ and $q \in \mathcal{P}os(t)$, then $(s[t]_p)|_{pq} = t|_q$.

Exercise 4.3 - Substitutions

[Points: 2+2+1]

For this exercise, we use infix notations for terms.

1. Let $t = \neg(x \wedge (\mathbf{T} \vee y)) \in T(\Sigma_{pred}, X)$. Compute $\sigma(t)$ and $\tau\sigma(t)$ where $\sigma = \{x \mapsto \mathbf{F}, y \mapsto \mathbf{T} \wedge x\}$ and $\tau = \{z \mapsto \mathbf{T}, x \mapsto \mathbf{T}\}$.
2. Let $t = \neg(\mathbf{T} \wedge (\mathbf{F} \vee y)) \in T(\Sigma_{pred}, X)$ and $s = \neg(x \wedge (\mathbf{F} \vee (x \wedge \mathbf{F}))) \in T(\Sigma_{pred}, X)$. Find a substitution σ such that $\sigma(t) = \sigma(s)$.
3. Is substitution composition commutative? If yes, give a proof. Otherwise give a counterexample.

Exercise 4.4 - Σ -Algebras

[Points: 1+1+2+1]

1. Explain why $\hat{\mathcal{J}}$ is required to be a total function.
2. Explain why $\hat{\mathcal{J}}$ is required to be unique.
3. Define the ADT **bool** for the boolean algebra with the function symbols **true**, **false**, **not**, **and**, **or**, including the usual identities.
4. Give an implementation for the ADT **bool**.