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Theory I, Assignment 5

http://lak.informatik.uni-freiburg.de/lak_teaching/ss_10/theory1.php

Submission: hand in by 08. Jul. 2010, before 4:00 p.m.

- The solutions should be submitted in English.
- You are encouraged to work in groups of two, and submit one solution.
- Your solution should be delivered to the lockbox in building 051 floor 00.

Exercise 5.1 - Stack ADT [Points: 1+1+1+1+1]

Specify an ADT $\mathsf{Stack}(A)$ for stacks. The operations available for this ADT should be as follows:

- new: Constructs a new empty stack.
- push: Adds an element to the top of the given stack.
- pop: Removes the top element of the given stack.
- top: Returns the top element of the given stack.
- isEmpty: Checks whether a given stack is empty.

Specify the signatures for these operations and define sensible identities for them. What are the constructors of the stack ADT?

Exercise 5.2 - ADT implementations [Points: 1+1+1+2]

Consider the following ADT:

$$\begin{aligned} \Sigma_\chi &= \{Z^{(0)}, F^{(1)}\} \\ \mathcal{E}_\chi &= \{F(F(x)) = x\} \end{aligned}$$

1. Prove that $[F(Z)]_{\mathcal{E}_\chi} = [F(F(F(Z)))]\mathcal{E}_\chi$
2. Give two Σ_χ-algebras such that all identities in $\mathcal{E}$ are valid in these algebras. The carrier set of the first Σ_χ-algebra should have one element, the carrier set of the second Σ_χ-algebra should have two elements.
3. How do the equivalence clases of $\approx_{\mathcal{E}}$ look like? Prove that your answer is correct.
4. Are the two Σ_χ-algebras given in 2 implementations in the revised sense? You need to verify that for $\Gamma = \Sigma_\chi$ and all $s, t \in T(\Gamma, \emptyset)$ we have $s \approx_{\mathcal{E}} t$ if and only if $\widehat{\alpha}(s) = \widehat{\alpha}(t)$.

Exercise 5.3 - Reductions [Points: 5]

Let f be a binary function symbol, and

$$\mathcal{E} = \{f(x, f(y, z)) = f(f(x, y), z), f(f(x, y), x) = x\}$$

Show that $f(x, x) \stackrel{*}{\longleftrightarrow}_{\mathcal{E}} x$. For every reduction step, give the identity, the term position, and the substitution (according to the definition of $\rightarrow_{\mathcal{E}}$) you have used.

Exercise 5.4 - Reductions [Points: 5]

A reduction relation $\rightarrow$ on a set M is called *normalizing* iff every element in M has a normal form. Find a normalizing reduction relation that is not terminating.