Albert-Ludwigs-Universität
Institut für Informatik
Prof. Dr. S. Albers
S. Lauer

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Algorithms Theory, Winter Term 07/08
Assignment 7

hand in by Monday, February 11, 2008, 14 p.m.
(boxes in building 051)

Exercise 1: Matrix chains (5 points)
Determine an optimal parenthesization for a chain of 4 matrices with dimensions $p = \langle 7, 10, 5, 8, 30\rangle$. Apply algorithm *dyn-mat-chain* and specify all values $m[i, j]$ and $s[i, j]$ for $1 \leq i \leq j \leq 4$. While determining the $m[i, j]$, show which expressions have to be considered in order to find the minimum value. Finally, show how the optimal parenthesization for the chain $\langle A_1, A_2, A_3, A_4\rangle$ looks like.

Exercise 2: Optimal binary search tree (5 points)
Suppose that for the keys $-\infty = k_0, k_1, k_2, k_3, k_4, k_5 = \infty$ the following request frequencies are known:

(k_0, k_1)	k_1	(k_1, k_2)	k_2	(k_2, k_3)	k_3	(k_3, k_4)	k_4	(k_4, k_5)
3	4	2	5	3	3	0	2	1

Create an optimal binary search tree using the approach from the lecture.

Proceed as follows: At first, specify the values $a_1, \ldots, a_4$ and $b_0, \ldots, b_4$ and fill in the tables

i	0	1	...	$4-h$
$W(i, i+h)$				
$P(i, i+h)$				
$r(i, i+h)$				

for $h = 0, \ldots, 4$. Then, starting with $k_{r(0,4)}$ as the root node, build up the resulting optimal binary search tree T. What is its weighted path length $P(T)$?

Exercise 3: Edit distance (5 points)
Consider the two strings $A =$ LASAGNA and $B =$ LANGUAGE.

a) Draw the trace graph for transforming A into B. For each node, draw in only those incoming edges that represent feasible transitions.

b) Mark an optimal trace (i.e. an optimal path in the trace graph) and specify the corresponding sequence of edit operations. What is the resulting edit distance $D(A, B)$?

Exercise 4: Ukkonen's algorithm (5 points)
Construct an explicit suffix tree for the string $t =$ REMEMBER using Ukkonen's algorithm: For each $i = 1, \ldots, |t|$, draw the implicit suffix tree T_i and specify the rules that have been applied in extensions $1, \ldots, i$. Finally, draw the explicit suffix tree T for the extended string REMEMBER\$ and insert all suffix links.