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Algorithms Theory, Assignment 5

http://lak.informatik.uni-freiburg.de/lak_teaching/ws09_10/algo0910.php

Submission: 14. Jan. 2010, 4 p.m.

- The solutions can be submitted in English or German
- You are required to submit your own solution.
- You are allowed to discuss your solutions with each other. Nevertheless, you are required to write down the answers in your own words.

Exercise 5.1 - Fibonacci Heaps [Points: 5]

Execute the following operations on an initialy empty Fibonacci heap:

$$insert(18), insert(14), insert(17), insert(28), insert(32),$$
$$insert(37), insert(25), insert(36), insert(53), insert(40),$$
$$deletemin(), decreasekey(40, 30), delete(36), deletemin().$$

For all intermediate steps, illustrate the resulting Fibonacci heap. New elements should always be inserted to the right of the current minimum. The consolidation operation after $deleteMin()$ starts with the next element on the right hand side of the deleted minimum.

Exercise 5.2 - Fibonacci Heaps [Points: 5]

Show that the following claim is *not* true:

> The maximum height of a tree within a Fibonacci Heap with n nodes is $\mathcal{O}(\log n)$.

Proceed as follows: For an arbitrary $n > 0$ give a sequence of operations that creates a Fibonacci heap finally consisting of one tree that is a linear chain of n nodes.

Exercise 5.3 - Disjoint-set forests [Points: 3+2]

Consider the implementation of disjoint sets, where sets are represented by rooted trees as in the lecture.

- Give a sequence of m $makeSet$, $union$, and $findSet$ operations, n of which are $makeSet$ operations, that require in total $\Theta(m \ \lg \ n)$ time. You are allowed to use union by rank, but findSet *without* path compression only.
- Give an iterative version of the $findSet$ procedure (with path compression).

Exercise 5.4 - Ackerman Function [Points: 5]

The Ackerman function $A : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ is defined as follows:

$$A(0, j) = j + 1$$
$$A(k, j) = A^{(j+1)}(k - 1, j) \qquad \text{for } k \geq 1$$
$$\text{where } A^{i+1}(k, j) := A(k, A^i(k, j)) \text{ and } A^1(k, j) = A(k, j)$$

Prove that $A(k + 1, j) \geq A(k, j)$ for any $k, j \geq 0$.