
- The solutions can be submitted in English or German
- You are required to submit your own solution.
- You are allowed to discuss your solutions with each other. Nevertheless, you are required to write down the answers in your own words.

**Exercise 5.1 - Fibonacci Heaps**

Execute the following operations on an initially empty Fibonacci heap:

\[ \text{insert}(18), \text{insert}(14), \text{insert}(17), \text{insert}(28), \text{insert}(32), \]
\[ \text{insert}(37), \text{insert}(25), \text{insert}(36), \text{insert}(53), \text{insert}(40), \]
\[ \text{deletemin}(), \text{decreasekey}(40, 30), \text{delete}(36), \text{deletemin}(). \]

For all intermediate steps, illustrate the resulting Fibonacci heap. New elements should always be inserted to the right of the current minimum. The consolidation operation after \( \text{deletemin}() \) starts with the next element on the right hand side of the deleted minimum.

**Exercise 5.2 - Fibonacci Heaps**

Show that the following claim is not true:

The maximum height of a tree within a Fibonacci Heap with \( n \) nodes is \( O(\log n) \).

Proceed as follows: For an arbitrary \( n > 0 \) give a sequence of operations that creates a Fibonacci heap finally consisting of one tree that is a linear chain of \( n \) nodes.

**Exercise 5.3 - Disjoint-set forests**

Consider the implementation of disjoint sets, where sets are represented by rooted trees as in the lecture.

- Give a sequence of \( m \text{ makeSet} \), \( \text{union} \), and \( \text{findSet} \) operations, \( n \) of which are \( \text{makeSet} \) operations, that require in total \( \Theta(m \log n) \) time. You are allowed to use union by rank, but \( \text{findSet} \) without path compression only.

- Give an iterative version of the \( \text{findSet} \) procedure (with path compression).

**Exercise 5.4 - Ackerman Function**

The Ackerman function \( A : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N} \) is defined as follows:

\[
A(0, j) = j + 1 \\
A(k, j) = A^{j+1}(k-1, j) \quad \text{for } k \geq 1 \\
\text{where } A^{i+1}(k, j) := A(k, A^i(k, j)) \text{ and } A^1(k, j) = A(k, j)
\]

Prove that \( A(k+1, j) \geq A(k, j) \) for any \( k, j \geq 0 \).