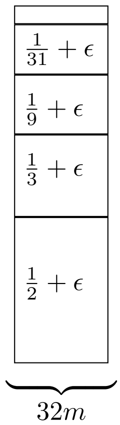
Algorithms Theory, Solution for Assignment 7

http://lak.informatik.uni-freiburg.de/lak_teaching/ws09_10/algo0910.php

Exercise 7.1 - Bin packing

- An optimal packing yields $\mathsf{OPT}(I) = 32m$ bins.



- The strategy First Fit Decreasing first sorts the sequence I:

$$\underbrace{\frac{1}{2} + \epsilon, \ldots, \frac{1}{2} + \epsilon}_{32m}, \underbrace{\frac{1}{3} + \epsilon, \ldots, \frac{1}{3} + \epsilon}_{32m}, \underbrace{\frac{1}{9} + \epsilon, \ldots, \frac{1}{9} + \epsilon}_{32m}, \underbrace{\frac{1}{31} + \epsilon, \ldots, \frac{1}{31} + \epsilon}_{32m}$$

  Then, it applies First Fit which results in the optimal packing showed above using $32m$ bins.

Exercise 7.2 - Matrix-chain multiplication

Find the optimal parenthesization of a matrix-chain product whose sequence P of dimensions is $\langle 44, 43, 3, 29, 35, 19\rangle$. Specify all values $m[i,j]$ and $s[i,j]$. Finally, provide the optimal parenthesization for $A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5$.
$P = \langle p_0, p_1, p_2, p_3, p_4, p_5\rangle$

- <u>l=1:</u>
  - $m[i,i] = 0 \quad i \in \{1,2,3,4,5\}$
- <u>l=2:</u>
  - $m[1,2] = min_{k=1}(m[1,k] + m[k+1,2] + p_0p_1p_2) = 5676$
  - $m[2,3] = min_{k=2}(m[2,k] + m[k+1,3] + p_1p_2p_3) = 3741$
  - $m[3,4] = min_{k=3}(m[3,k] + m[k+1,4] + p_2P_3p_4) = 3045$
  - $m[4,5] = min_{k=4}(m[4,k] + m[k+1,5] + p_3p_4p_5) = 19285$
- <u>l=3:</u>
  - $m[1,3] = min_{k=1,2}(m[1,k] + m[k+1,3] + p_0p_kp_3)$
    $m[1,3] = min \left\{ \begin{array}{l} m[1,1] + m[2,3] + p_0p_1p_3 = 0 + 3741 + 54868 = 58609 \\ m[1,2] + m[3,3] + p_0p_2p_3 = 5676 + 0 + 3828 = 9504 \end{array} \right. = 9504$
    $s[1,3] = 2$

- $m[2,4] = min_{k=2,3}(m[2,k] + m[k+1,4] + p_1 p_k p_4)$
  $$m[2,4] = min \begin{cases} m[2,2] + m[3,4] + p_1p_2p_4 = 0 + 3045 + 4515 = 7560 \\ m[2,3] + m[4,4] + p_1p_3p_4 = 3741 + 0 + 43645 = 47386 \end{cases} = 7560$$
  $s[2,4] = 2$
- $m[3,5] = min_{k=3,4}(m[3,k] + m[k+1,5] + p_2 p_k p_5)$
  $$m[3,5] = min \begin{cases} m[3,3] + m[4,5] + p_2p_3p_5 = 0 + 19285 + 1653 = 20938 \\ m[3,4] + m[5,5] + p_2p_4p_5 = 3045 + 0 + 1995 = 5040 \end{cases} = 5040$$
  $s[3,5] = 4$

- <u>l=4:</u>
  - $m[1,4] = min_{k=1,2,3}(m[1,k] + m[k+1,4] + p_0 p_k p_4)$
    $$m[1,4] = min \begin{cases} m[1,1] + m[2,4] + p_0p_1p_4 = 0 + 7560 + 66220 = 73780 \\ m[1,2] + m[3,4] + p_0p_2p_4 = 5676 + 3045 + 4620 = 13341 \\ m[1,3] + m[4,4] + p_0p_3p_4 = 9504 + 0 + 44660 = 54164 \end{cases} = 13341$$
    $s[1,4] = 2$
  - $m[2,5] = min_{k=2,3,4}(m[2,k] + m[k+1,5] + p_1 p_k p_5)$
    $$m[2,5] = min \begin{cases} m[2,2] + m[3,5] + p_1p_2p_5 = 0 + 5040 + 2451 = 7491 \\ m[2,3] + m[4,5] + p_1p_3p_5 = 3741 + 19285 + 23693 = 46719 \\ m[2,4] + m[5,5] + p_1p_4p_5 = 7560 + 0 + 28595 = 36155 \end{cases} = 7491$$
    $s[2,5] = 2$

- <u>l=4:</u>
  - $m[1,5] = min_{k=1,2,3,4}(m[1,k] + m[k+1,5] + p_0 p_k p_5)$
    $$m[1,5] = min \begin{cases} m[1,1] + m[2,5] + p_0p_1p_5 = 0 + 7491 + 35948 = 43439 \\ m[1,2] + m[3,5] + p_0p_2p_5 = 5676 + 5040 + 2508 = 13224 \\ m[1,3] + m[4,5] + p_0p_3p_5 = 9504 + 19285 + 24244 = 53033 \\ m[1,4] + m[5,5] + p_0p_4p_5 = 13341 + 0 + 29260 = 42601 \end{cases} = 13224$$
    $s[1,5] = 2$

Resulting tables:

m (i \ j)	1	2	3	4	5
1	0	5676	9504	13341	13224
2		0	3741	7560	7491
3			0	3045	5040
4				0	19285
5					0

s (i \ j)	2	3	4	5
1	1	2	2	2
2		2	2	2
3			3	4
4				4

Final parenthesization: $(A_1A_2)((A_3A_4)A_5)$

Exercise 7.3 - Optimal search trees

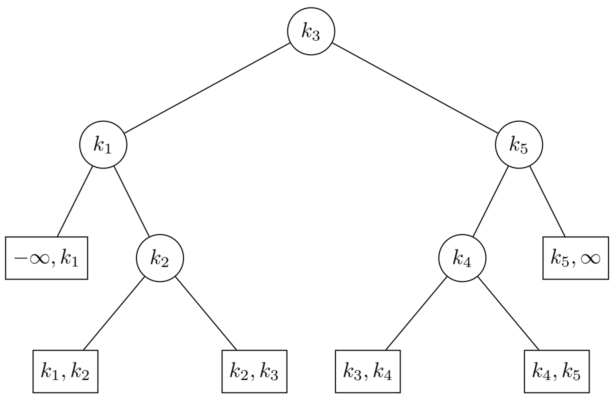
Applying the algorithm we optain the following tables:

	W	j = 0	1	2	3	4	5
	0	4	12	17	25	35	43
i	1		3	8	16	26	34
	2			2	10	20	28
	3				1	11	19
	4					4	12
	5						3

	P	j = 0	1	2	3	4	5
	0	0	12	25	47	71	98
i	1		0	8	24	45	70
	2			0	10	30	50
	3				0	11	30
	4					0	12
	5						0

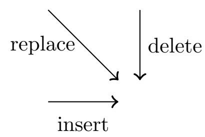
	r	j = 1	2	3	4	5
	0	1	1	2	3	3
i	1		2	3	3	4
	2			3	4	4
	3				4	5
	4					5

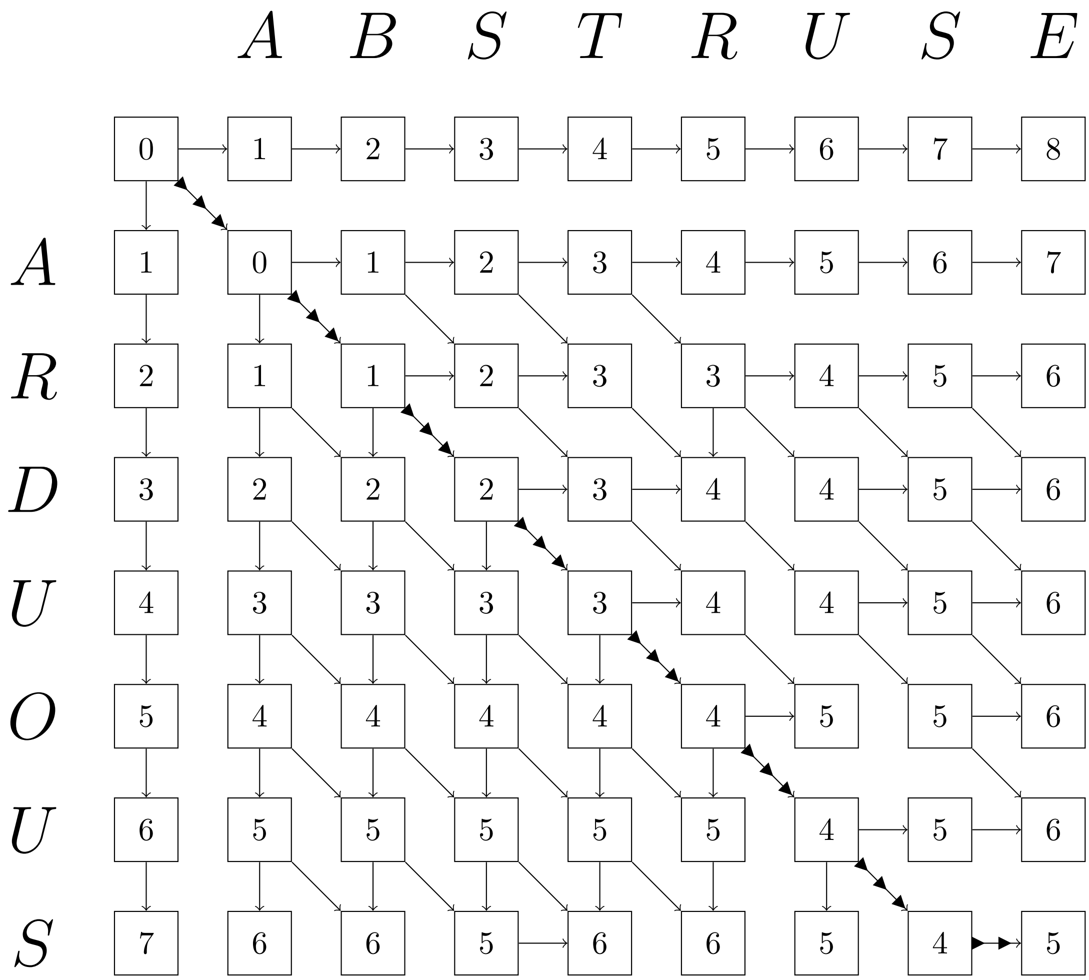
Optimal search tree:



Exercise 7.4 - Edit Distance

- Trace graph:



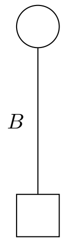


- Sequence: $D(A, B) = 5$

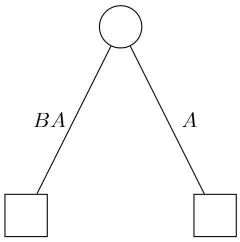
Operation	Cost
replace A by A	0
replace R by B	1
replace D by S	1
replace U by T	1
replace O by R	1
replace U by U	0
replace S by S	0
insert E	1

Exercise 7.5 - Text search (Ukkonen algorithm)

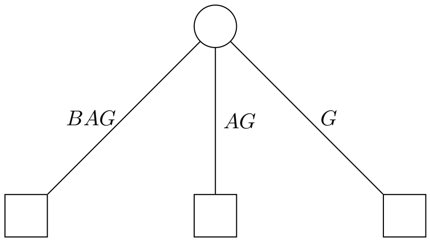
- T_1 :
  - rule 2



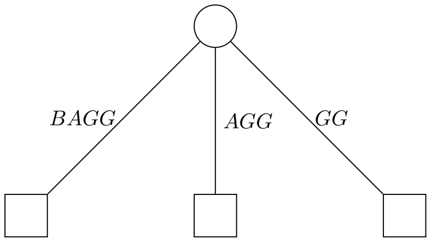
- T_2 :
  - apply implicit extensions
  - ext 2, rule 2



- T_3 :
  - apply implicit extensions
  - ext 3, rule 2

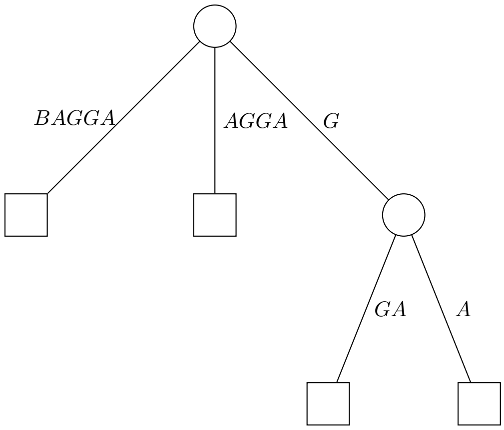


- T_4 :
  - apply implicit extensions
  - ext 4, rule 3

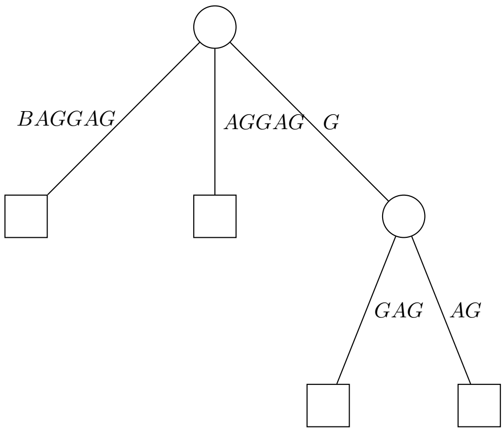


- T_5 :
  - apply implicit extensions

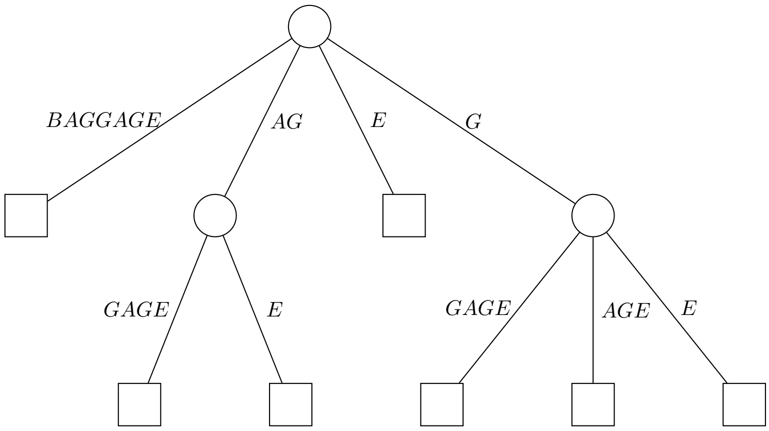
- ext 4, rule 2
- ext 5, rule 3



- $\underline{T_6:}$
  - apply implicit extensions
  - ext 5, rule 3, phase ended



- $\underline{T_7:}$
  - apply implicit extensions
  - ext 5, rule 2
  - ext 6, rule 2
  - ext 7, rule 2



- $\underline{T} : BAGGAGE$