Network Algorithms, Summer Term 2014
Problem Set 5

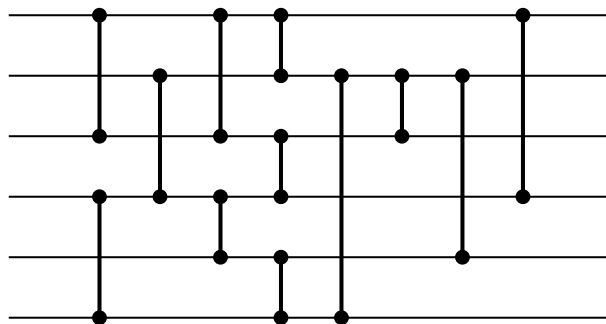## Exercise 1: Sorting Networks

Figure 1: A Sorting Network?

For each of the following questions, prove or disprove the given claim.

Hint: Whenever you need to construct a network as counterexample, three wires will suffice.

1. The network of 6 wires and 12 comparators in Figure 1 above is a sorting network, that is, it sorts each input sequence of numbers correctly.

2. Given any correct sorting network, adding another comparator at the end can destroy the sorting property.

3. Given any correct sorting network, adding another comparator at the front can **not** destroy the sorting property.

4. Every correct sorting network needs to have at least one comparator between each two **consecutive** wires.

5. A network which contains all $\binom{n}{2}$ comparators between any two of the n wires, in whatever order they are placed, is a correct sorting network.

6. Given any correct sorting network, adding another comparator anywhere does not destroy the sorting property.

7. Given any correct sorting network, inverting it (i.e., feeding the input into the output wires and traversing the network "from right to left") results in another correct sorting network.