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# Algorithms and Datastructures
# Summer Term 2024
# Exercise Sheet 2

**Due:** Thursday, May 1st, 2pm

## Exercise 1: $\mathcal{O}$-notation *(9 Points)*

Prove or disprove the following statements. Use the *set definition* of the $\mathcal{O}$-notation (lecture slides week 2, slides 11 and 12).

(a) $2n^3 + 4n^2 + 7\sqrt{n} \in \mathcal{O}(n^3)$ *(1 Point)*

(b) $n \cdot \log_3(n) \in \omega(n \cdot \log_5(n))$ *(2 Points)*

(c) $2^n \in o(n!)$ *(2 Points)*

(d) $2\log_2\left(n^2\right) \in \Omega((\log_2 n)^2)$ *(2 Points)*

(e) $\max\{f(n), g(n)\} \in \Theta(f(n) + g(n))$ for non-negative functions $f$ and $g$. *(2 Points)*

## Exercise 2: Sorting by asymptotic growth *(4 Points)*

Sort the following functions by their asymptotic growth. Write $g <_{\mathcal{O}} f$ if $g \in \mathcal{O}(f)$ *and* $f \notin \mathcal{O}(g)$. Write $g =_{\mathcal{O}} f$ if $f \in \mathcal{O}(g)$ *and* $g \in \mathcal{O}(f)$ (no proof needed).

| | | | |
|---|---|---|---|
| $\sqrt{n}$ | $2^n$ | $n!$ | $\log(n^3)$ |
| $3^n$ | $n^{100}$ | $\log(\sqrt{n})$ | $(\log n)^2$ |
| $\log n$ | $10^{100}n$ | $(n+1)!$ | $n \log n$ |
| $2^{(n^2)}$ | $n^n$ | $\sqrt{\log n}$ | $(2^n)^2$ |

## Exercise 3: Stable Sorting *(7 Points)*

A sorting algorithm is called stable if elements with the same key remain in the same order. E.g., assume you want to sort the following tuples with respect to their integer key:

$$[(3, \textit{"blue"}), (1, \textit{"green"}), (1, \text{"}red\text{"}), (7, \text{"}gray\text{"}), (4, \text{"}yellow\text{"}), (3, \text{"}orange\text{"}), (4, \textit{"white"}), (3, \text{"}black\text{"})]$$

A *stable* sorting algorithm must generate the following output:

$$[(1, \textit{"green"}), (1, \text{"}red\text{"}), (3, \textit{"blue"}), (3, \text{"}orange\text{"}), (3, \text{"}black\text{"}), (4, \text{"}yellow\text{"}), (4, \textit{"white"}), (7, \text{"}gray\text{"})]$$

A sorting algorithm is not stable (with respect to the sorting key) if it outputs, e.g., the following:

$$[(1, \textit{"red"}), (1, \textit{"green"}), (3, \textit{"black"}), (3, \text{"}blue\text{"}), (3, \text{"}orange\text{"}), (4, \text{"}yellow\text{"}), (4, \textit{"white"}), (7, \text{"}gray\text{"})]$$

(a) Give an example that shows that `QuickSort` is not stable. *(2 Points)*

(b) Describe a method to make any *comparison-based* sorting algorithm stable, without changing the *asymptotic* runtime. Explain. *(5 Points)*