# Theoretical Computer Science - Bridging Course Exercise Sheet 2 

Due: Tuesday, 30th of April 2024, 12:00 pm

## Exercise 1: Constructing DFAs, NFAs

(2+2+2 Points)
Construct DFAs that recognize the first two languages and an NFA that recognizes the last language. The alphabet set is $\Sigma=\{a, b\}$.

1. $L_{1}=\{w \mid w$ has an odd number of $a$ 's and ends with $b\}$.
2. $L_{2}=\{w \mid w$ is any string except $b b$ and $b b b\}$.
3. $L_{3}=\{w \mid w$ is any string where at least one of the symbols $a$ or $b$ occurs an even number of times $\}$.

## Exercise 2: Closure of Regular Languages

(2+3+2+2 Points)

1. Show that if $M$ is a DFA that recognizes language $L$, you can construct a new DFA $M^{\prime}$ that recognizes the complement of $L$ i.e. $\bar{L}:=\Sigma^{*} \backslash L$. Conclude that the class of regular languages is closed under complementation.
2. Show by giving an example that if $M$ is an NFA (instead of a DFA) that recognizes language $L$, then the same approach you used to construct the new DFA $M^{\prime}$ above doesn't necessarily yield a new NFA that recognizes the complement of $L$. Is the class of languages recognized by NFAs closed under complementation? Explain your answer.

Let $L_{1}$ and $L_{2}$ be regular languages.
3. Show that $L_{1} \cap L_{2}$ is regular by constructing its corresponding DFA.
4. Deduce from parts 1 and 3 that regular languages are closed under the symmetric difference i.e. $L_{1} \Delta L_{2}$ is also regular.

Remark: For parts 1 and 3 there's no need for drawing state diagrams. Show how a DFA for the language in the question can be constructed presuming the existence of DFAs for $L, L_{1}$, and $L_{2}$.

## Exercise 3: NFA to DFA

Consider the following NFA.


1. Give a formal description of the NFA by giving the alphabet, state set, transition function, start state and the set of accept states.
2. Construct a DFA which is equivalent to the above NFA by drawing the corresponding state diagram.

Bonus question: Explain which language the automaton accepts.

