Albert-Ludwigs-Universität, Inst. für Informatik
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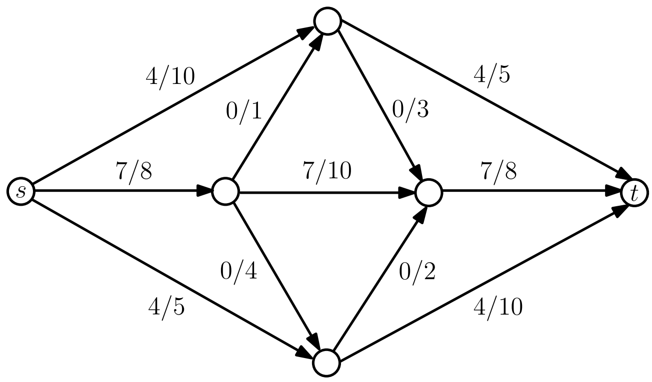
Algorithm Theory, Winter Term 2015/16
Problem Set 7

hand in (hard copy or electronically) by 10:15, Thursday December 10, 2015, tutorial session will be on December 14, 2015

Exercise 1: Ford-Fulkerson (5+1+2 points)

Consider the following flow network, where for each edge, the capacity (second larger number) and a current flow value (first smaller number) are given.



a) Find a maximal flow in the given network with the help of the Ford Fulkerson algorithm. Draw the residual graph with all the residual capacities in all steps.

b) Is the following statement true or false? *If all edges in a flow network have distinct capacities, then there is a unique maximum flow.* Justify your answer with a (short) proof or give a counterexample.

c) You are given a (connected) directed graph $G = (V, E)$, with positive integer capacities on each edge, a designated source $s \in V$, and a designated sink $t \in V$. Additionally you are given a current maximum $s - t$ flow $f : E \to \mathbb{R}_{\geq 0}$.

   Now suppose we increase the capacity of one specific edge $e_0 \in E$ by one unit. Show how to find a maximum flow in the resulting capacitated graph in time $\mathcal{O}(|E|)$.

Exercise 2: Seating arrangement (4 points)

A group of international students go out to eat dinner together. To increase their social interaction, they would like to sit at tables so that no two students from the same country are at the same table. Show how to formulate finding a seating arrangement that meets this objective as a maximum flow problem. Assume that the students are from p different countries and there are s_i students from the i^{th} country. Also assume that q tables are available and that the j^{th} table has a seating capacity of t_j.