Algorithm Theory

Chapter 6
Graph Algorithms

Part V:
Baseball Elimination

Fabian Kuhn

Baseball Elimination

Team i	Wins w_i	Losses ℓ_i	To Play r_i	Against = r_{ij}				
				NY	Balt.	T. Bay	Tor.	Bost.
New York	81	69	12	-	2	5	2	3
Baltimore	79	77	6	2	-	2	1	1
Tampa Bay	79	74	9	5	2	-	1	1
Toronto	76	80	6	2	1	1	-	2
Boston	71	84	7	3	1	1	2	-

- Only wins/losses possible (no ties), winner: team with most wins
- Which teams can still win (as least as many wins as top team)?
- Boston is eliminated (cannot win):
  - Boston can get at most 78 wins, New York already has 81 wins
- If for some i, j: $w_i + r_i < w_j$ → team i is eliminated
- Sufficient condition, but not a necessary one!

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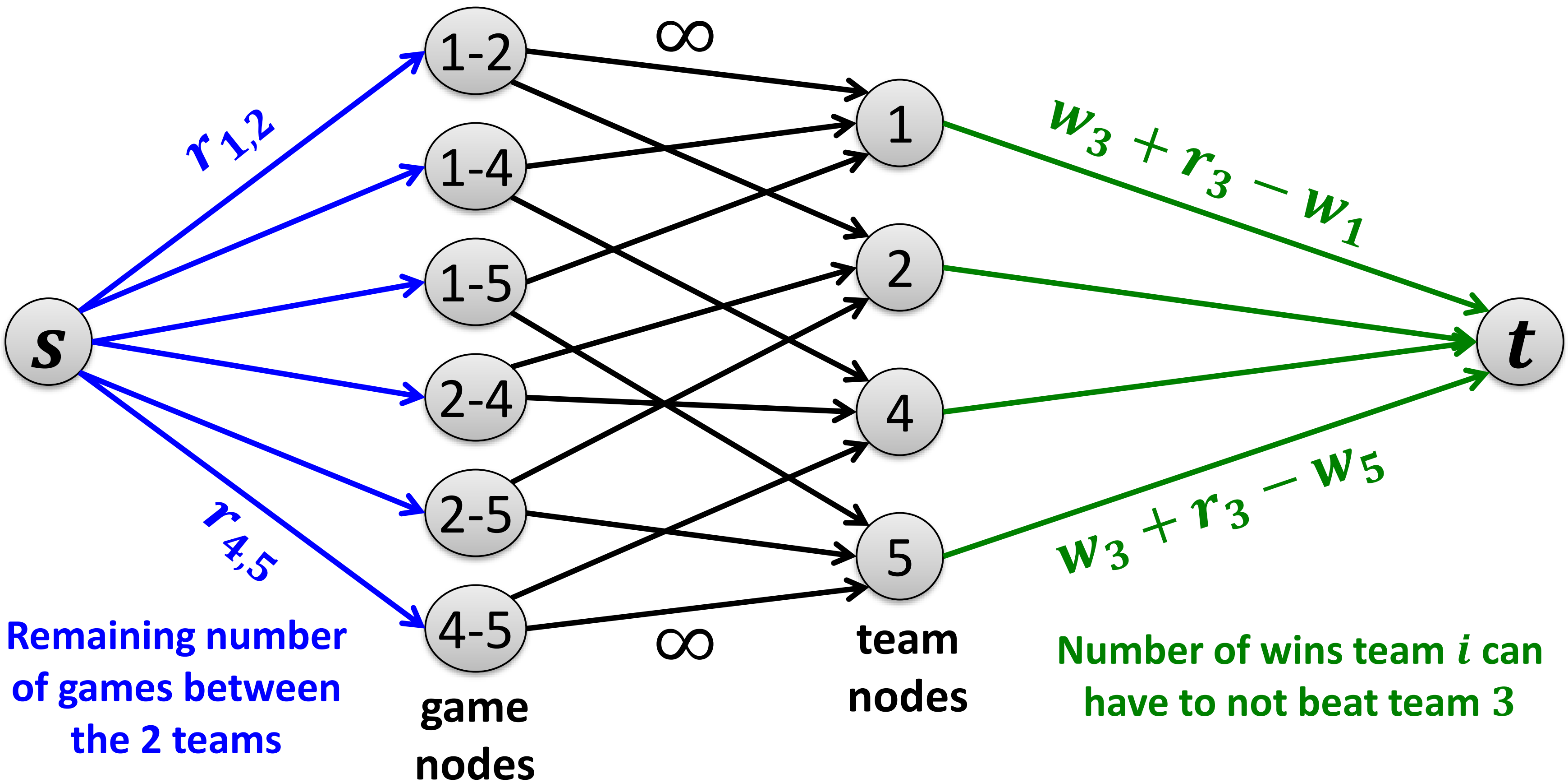
- Can Toronto still finish first?
- Toronto can get $82 > 81$ wins, but:
  NY and Tampa have to play 5 more times against each other
  → if NY wins two, it gets 83 wins, otherwise, Tampa has 83 wins
- Hence: Toronto cannot finish first
- How about the others? How can we solve this in general?

Max Flow Formulation

- Can team 3 finish with most wins?



- Team 3 can finish first iff all source-game edges are saturated

Reason for Elimination

AL East: Aug 30, 1996

Team i	Wins w_i	Losses ℓ_i	To Play r_i	Against = r_{ij}				
				NY	Balt.	Bost.	Tor.	Detr.
New York	75	59	28	-	3	8	7	3
Baltimore	71	63	28	3	-	2	7	4
Boston	69	66	27	8	2	-	0	0
Toronto	63	72	27	7	7	0	-	0
Detroit	49	86	27	3	4	0	0	-

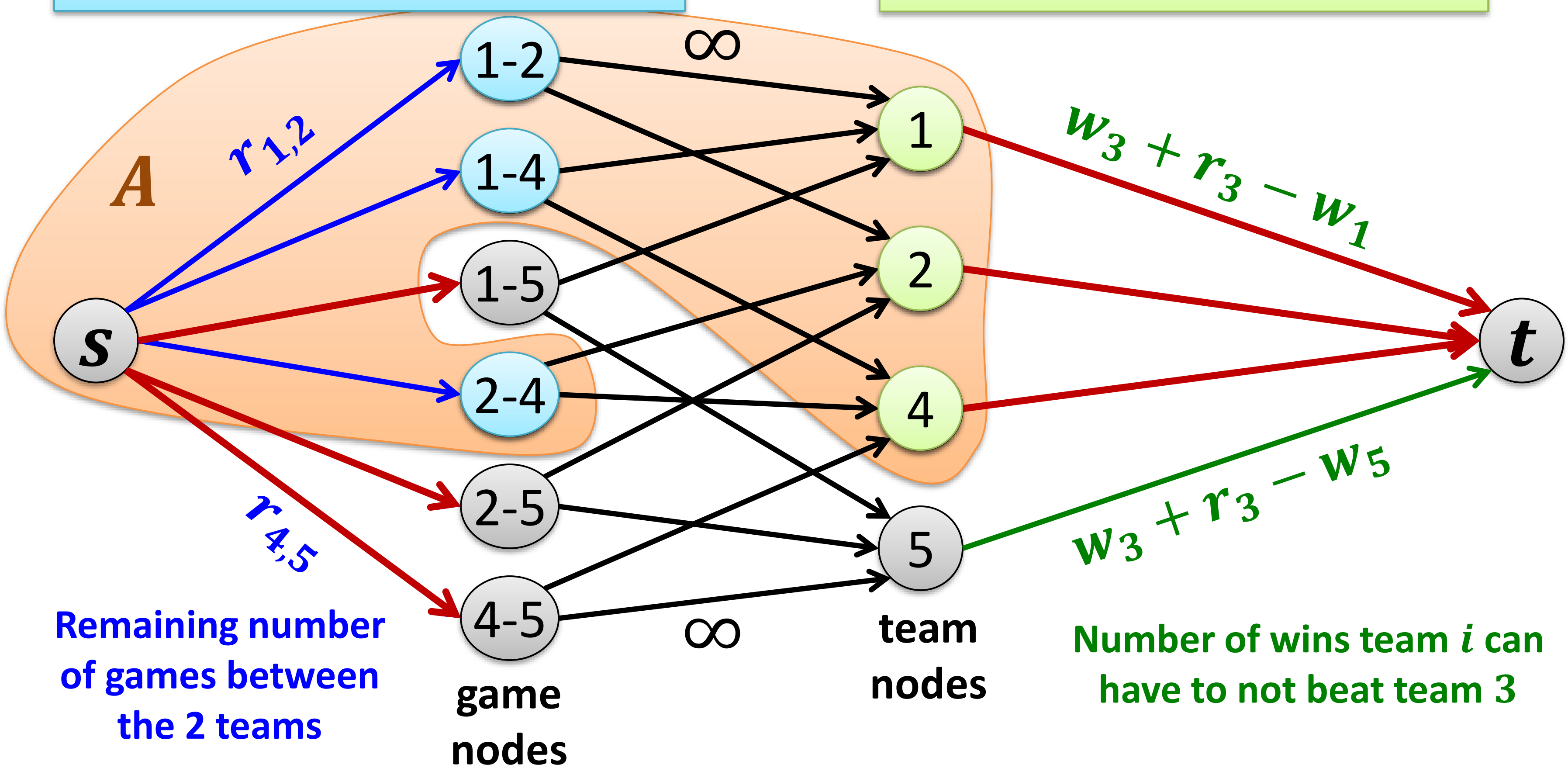
- Detroit could finish with $49 + 27 = 76$ wins
- Consider $R = \{\mathrm{NY}, \mathrm{Bal}, \mathrm{Bos}, \mathrm{Tor}\}$
  - Have together already won $w(R) = 278$ games
  - Must together win at least $r(R) = 27$ more games
- On average, teams in R win $\frac{278+27}{4} = 76.25$ games

Reason for Elimination

Team 3 eliminated $\Leftrightarrow$ min cut $(A, V \setminus A)$ of cap. < "all blue edges"

A contains all game nodes for teams in R

A contains team nodes R with $R \neq \emptyset$



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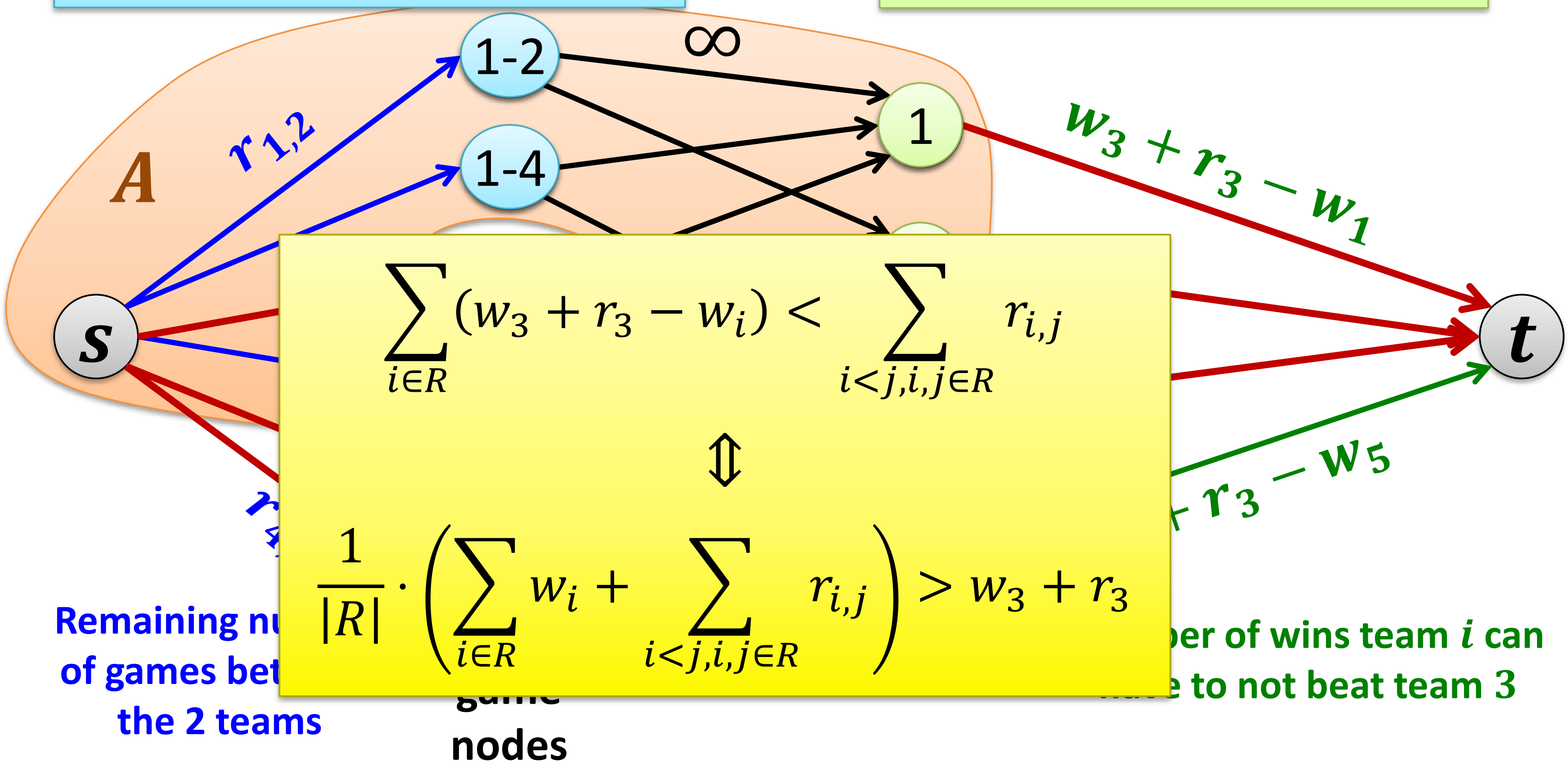

$$\sum_{i \in R} (w_3 + r_3 - w_i) < \sum_{i<j, i,j \in R} r_{i,j}$$

$$\Updownarrow$$

$$\frac{1}{|R|} \cdot \left(\sum_{i \in R} w_i + \sum_{i<j, i,j \in R} r_{i,j} \right) > w_3 + r_3$$

Reason for Elimination

Certificate of elimination:

$$R \subseteq X, \qquad w(R) := \underbrace{\sum_{i \in R} w_i}_{\text{#wins of nodes in } R}, \qquad r(R) := \underbrace{\sum_{i,j \in R} r_{i,j}}_{\text{#remaining games among nodes in } R}$$

#wins of nodes in R

#remaining games among nodes in R

- Team $x \in X$ is eliminated by $R \subseteq X \setminus \{x\}$ if

$$\frac{w(R) + r(R)}{|R|} > w_x + r_x.$$

- If team $x \in X$ is eliminated, there exists $R \subseteq X \setminus \{x\}$ such that team x is eliminated by R.
  – R can be constructed by looking at a minimum cut