University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
S. Faour

Theoretical Computer Science - Bridging Course
Exercise Sheet 11

Due: Sunday, 6th of February 2022, 23:59 pm

Exercise 1: Pumping Lemma for Context-Free Languages

Let $L = \{w \in \Sigma^* \mid$ number of 1s equals number of 2s, and number of 3s equals number of 4s in $w\}$. Here, $\Sigma = \{1, 2, 3, 4\}$. Show that L is not context-free.

Exercise 2: Complexity Classes: Big Picture

(a) Why is $\mathcal{P} \subseteq \mathcal{NP}$?

(b) Show that $\mathcal{P} \cap \mathcal{NPC} = \emptyset$ if $\mathcal{P} \neq \mathcal{NP}$.
Hint: Assume that there exists a $L \in \mathcal{P} \cap \mathcal{NPC}$ and derive a contradiction to $\mathcal{P} \neq \mathcal{NP}$.

(c) Give a Venn Diagram showing the sets $\mathcal{P}, \mathcal{NP}, \mathcal{NPC}$ for both cases $\mathcal{P} \neq \mathcal{NP}$ and $\mathcal{P} = \mathcal{NP}$.
Remark: Use the results of (a) and (b) even if you did not succeed in proving those.

Exercise 3: Class $\mathcal{NPC}$

Show DominatingSet := $\{\langle G, k\rangle \mid$ Graph G has a dominating set of size at most $k\} \in \mathcal{NPC}$.
Use that VertexCover := $\{\langle G, k\rangle \mid$ Graph G has a vertex cover of size at most $k\} \in \mathcal{NPC}$.
*Remark: A **dominating set** is a subset of nodes of G such that every node not in the subset is adjacent to some node in the subset. A **vertex cover** is a subset of nodes of G such that every edge of G is incident to a node in the subset.*
Hint: Transform a Graph G into a Graph G' such that a vertex cover of G will result in a dominating set G' and vice versa(!). Note that a dominating set is not necessarily a vertex cover ($G = (\{v_1, v_2, v_3, v_4\}, \{\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}\})$ has the dominating set $\{v_1, v_4\}$ which is not a vertex cover). Also a vertex cover is not necessarily a dominating set (consider isolated notes).