University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
S. Faour

Theoretical Computer Science - Bridging Course
Exercise Sheet 3

Due: Sunday, 14th of October 2021, 23:59 pm

Exercise 1: Regular Expressions *(1+2+1+2 Points)*

Consider the following regular expressions. What language do they recognize? Give two strings that are members of the the corresponding language and two strings which are not members – a total of four strings for each part. Assume for the first two parts that the alphabet $\Sigma = \{a, b, c\}$.

(a) $a^*b^*c^*$

(b) $((a \cup c)^*b(a \cup c)^*b(a \cup c)^*b(a \cup c)^*)^*$

Give a regular expression for each of the following languages.

(c) L_1 is the language, over alphabet $\{a, b\}$, of all strings starting and ending with the same symbol from the alphabet Σ.

(d) L_2 is the language, over alphabet $\{0, 1\}$, of all alternating 0 and 1 strings of length at least 2.

Exercise 2: Limits of the Pumping Lemma *(2+3 Points)*

Consider the language $L = \{c^m a^n b^n \mid m, n \geq 0\} \cup \{a, b\}^*$ over the alphabet $\Sigma = \{a, b, c\}$.

(a) Describe in words (not using the pumping lemma), why L can not be a regular language.

(b) Show that, while the property described in the Pumping Lemma is a necessary condition for regularity, it is *not* sufficient for regularity.

Hint: Use L as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.

Exercise 3: Applications of the Pumping Lemma *(3+2 Points)*

Use the Pumping Lemma to show that the following languages over the alphabet $\Sigma = \{0, 1\}$ are not regular.

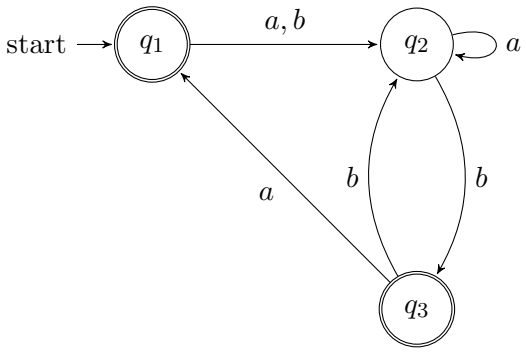
(a) $L = \{0^k \mid k \text{ is prime}\}$

(b) $L = \{0^k 1 w 0^k | k \geq 1 \text{ and } w \in \Sigma^*\}$.

Bonus part: Is the language $L = \{0^k w 0^k | k \geq 1 \text{ and } w \in \Sigma^*\}$ regular?

Exercise 4: NFA-GNFA-Regular Expression *(4 Points)*

Consider the following NFA:



Give the regular expression defining the language recognized by this NFA by converting it *stepwise* into an equivalent GNFA with only two nodes. Document your steps.