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Theoretical Computer Science - Bridging Course
Exercise Sheet 7

Due: Sunday, 19th of December 2021, 23:59 pm

Exercise 1: $\mathcal{O}$-Notation Formal Proofs *(1+2+3 Points)*

Roughly speaking, the set $\mathcal{O}(f)$ contains all functions that are not growing faster than the function f when additive or multiplicative constants are neglected. Formally:

$$g \in \mathcal{O}(f) \iff \exists c \geq 0, \exists M \in \mathbb{N}, \forall n \geq M : g(n) \leq c \cdot f(n)$$

For the following pairs of functions, state whether $f \in \mathcal{O}(g)$ or $g \in \mathcal{O}(f)$ or both. Proof your claims (you do not have to prove a negative result $\notin$, though).

(a) $f(n) = 100n,\ g(n) = 0.1 \cdot n^2$

(b) $f(n) = \sqrt[3]{n^2},\ g(n) = \sqrt{n}$

(c) $f(n) = \log_2(2^n \cdot n^3),\ g(n) = 3n$ *Hint: You may use that* $\log_2 n \leq n$ *for all* $n \in \mathbb{N}$.

Exercise 2: Sort Functions by Asymptotic Growth *(5 Points)*

Give a sequence of the following functions sorted by asymptotic growth, i.e., for consecutive functions g, f in your sequence, it should hold $g \in \mathcal{O}(f)$. Write "$g \cong f$" if $f \in \mathcal{O}(g)$ *and* $g \in \mathcal{O}(f)$.

$\log_2(n!)$	$\sqrt{n}$	2^n	$\log_2(n^2)$
3^n	n^{100}	$\log_2(\sqrt{n})$	$(\log_2 n)^2$
$\log_{10} n$	$10^{100} \cdot n$	$n!$	$n \log_2 n$
$n \cdot 2^n$	n^n	$\sqrt{\log_2 n}$	n^2

Exercise 3: The class $\mathcal{P}$ *(1+2+3+3 Points)*

Show that the following languages ($\cong$ problems) are in the class $\mathcal{P}$ by giving an algorithm that requires polynomial time in the input size. Use the $\mathcal{O}$-notation to bound the run-time of your algorithm. Since it is relatively easy (i.e., feasible in polynomial time) to decide whether an input is well-formed, your algorithm only needs to consider well-formed inputs.

(a) PALINDROME $:= \{w \in \{0,1\}^* \mid w$ is a Palindrome$\}$

(b) LIST $:= \{\langle A, c\rangle \mid A$ is a list of numbers that contains x,y such that $x + y = c\}$.

(c) 3-CLIQUE $:= \{\langle G\rangle \mid G$ has a *clique* of size at least 3$\}$

(d) 17-DOMINATINGSET $:= \{\langle G\rangle \mid G$ has a *dominating set* of size at most 17$\}$

Remarks:

- You may assume that $\langle A\rangle$ is an array.

- A *dominating set* of a graph $G = (V, E)$ is a subset $D \subseteq V$ such that for every vertex $v \in V$: $v \in D$ or v adjacent to a node $u \in D$.
- A *clique* of a graph $G = (V, E)$ is a subset $Q \subseteq V$ such that for all $u, v \in Q : \{u, v\} \in E$.