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Algorithm Theory
Exercise Sheet 9

Due: Wednesday, 11th of January, 2023, 11:59 pm

Exercise 1: Only Forward Ford Fulkerson *(5 Points)*

In the Ford Fulkerson algorithm we maintain a residual network and search an augmenting path in this network. The important difference is, that in the residual networks we have backwards edges that allow us to also reduce the amount of flow that is going through a given edge. In this exercise we want to show that it is absolutely necessary to have these backwards edges.

Consider an altered version of the Ford Fulkerson algorithm, that will not include backwards edges in the residual network. Show that this version of the Ford Fulkerson algorithm can't even give a constant approximation guarantee for the maximum flow problem. This means for any constant $C \in \mathbb{N}$ there exists a problem instance on which the algorithm outputs a solution that is at most $\frac{1}{C}$-times the optimal solution.

Exercise 2: Oracle Ford Fulkerson (updated bonus task) *(5 Points)*

Consider an execution of Ford Fulkerson, but instead of searching an augmenting path by some sort of graph traversal, we instead can ask an oracle that will give us some "perfect" augmenting path. Show that there is a set of augmenting paths the oracle can propose, such that in total at most $m = |E|$ iterations/ calls to the oracle are needed.

Exercise 3: Perfect Matchings in regular Graphs *(10 Points)*

We call an undirected Graph d-regular if each node has exactly d edges. Let $G = (A \cup B, E)$ be a bipartite d-regular graph.
Show that E is the union of d perfect matchings, i.e., show that $E = E_1 \dot\cup \ldots \dot\cup E_d$, where $E_1, \ldots, E_d$ are perfect matchings in G.